EM I: Electrostatics

FIZIKA SJPO Training

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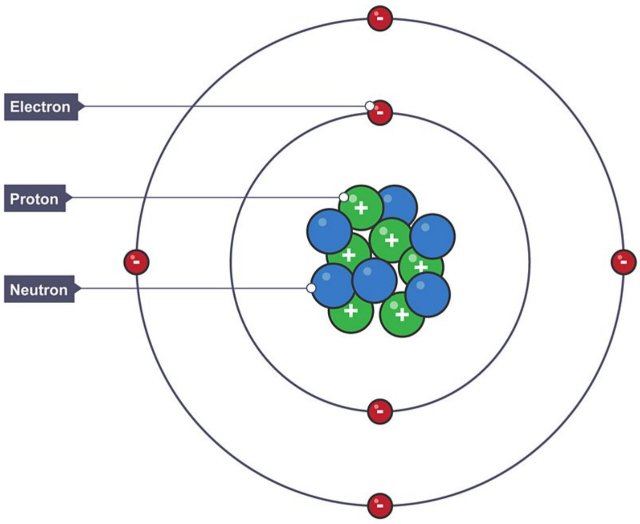
1.1 Electrostatics

Electrostatics is the study of forces, energy and motion of stationary, static charges.

1.1.1 Electric Charge

The **electric charge** (unit: C) is the fundamental building block of electrostatics. There are two types of charge - **positive** and **negative.** As you might have known, **like charges repel, unlike charges attract.**

In chemistry, you might have learnt about the **atomic structure.** Everything is made of atoms, and all atoms have three fundamental particles - the negatively-charged **electron,** the positively-charged **proton,** and the neutrally-charged **neutron.**



We say that the charges of the electron and the proton respectively are $-e$ and $+e$, where $e = 1.60 \times 10^{-19}$ C is the fundamental charge (a constant).

The principle of **conservation of charge** states that charge cannot be created nor destroyed. Hence, for any closed system, the sum of all electric charges must be constant.

The principle of **quantisation of charge** states that all charges in nature always occur as integer multiples of e. Charge is said to be *quantised*, so any charge $q = Ne$ where $N \in \mathbb{Z}$.

Example 1.1. Three identical metal balls A, B and C are mounted on insulating rods. Ball A has a charge of $+Q$ and balls B and C are initially uncharged. Ball A is touched first to ball B and then separately to ball C. What are the final charges on all of the balls?

Solution. By the principle of conservation of charge, through each of the steps, the total charge on balls A, B and C must be the same. Let's use a triplet $(+Q, 0, 0)$ to represent the charges on balls A, B and C.

After ball A touches ball B, by symmetry, ball A and B will each have the same charge (half of the total charge between balls A and B). Hence, the charges become $\left(+\frac{1}{2}Q, +\frac{1}{2}Q, 0\right)$. By the same logic, after ball A touches ball C, ball A and C will each have the same charge (half of the total charge between balls A and C). Hence, the charges become $\left(+\frac{1}{4}Q, +\frac{1}{2}Q, +\frac{1}{4}Q\right)$. These are the final charges on balls A, B and C respectively.

1.1.2 Electric Force & Field

Consider two point charges q_1 and q_2, located at a distance of r apart. The magnitude of the **electric force $\boldsymbol{F}_E$** between the two charges is given by **Coulomb's Law:**

$$F_E = \frac{k\,|q_1 q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} \tag{1}$$

The direction of $\boldsymbol{F}_E$ is either directly towards or directly away between the charges, depending on the signs of the charges (whether they attract or repel).

Here, $k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ is Coulomb's constant, and $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ is the permittivity of free space.

Remark. If both electric forces and gravitational forces are relevant in the same problem, we usually **treat gravitational forces as negligible** (unless otherwise stated).

The **electric field $\boldsymbol{E}$** can be thought of as the electric force per unit charge. Hence, for a point charge Q, the magnitude of $\boldsymbol{E}$ at a distance r away is given by:

$$E = \frac{k\,|Q|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} \tag{2}$$

The direction of $\boldsymbol{E}$ is either radially outwards (if Q is positive) or radially inwards (if Q is negative) from the charge.

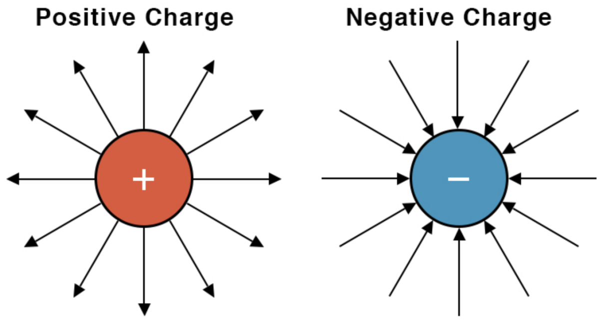
As such, if a charge q is present in an external electric field $\boldsymbol{E}$, it will experience an electric force $\boldsymbol{F}_\mathrm{E}$, whereby:

$$\boldsymbol{F}_E = q\boldsymbol{E} \tag{3}$$

The **principle of superposition** for electric forces states that the resultant force on any charge is the **vector sum** of the forces exerted by all the other individual charges. This just means that you can add up electric forces (and hence, electric fields) as vectors.

Remark. The sign of q needs to be taken into account for Equation (3). If q is positive, then the electric force is in the same direction as the electric field. If q is negative, then they are in opposite directions.

To visually represent electric field, we make use of **electric field lines.** These field lines also represent the direction of the electric force experienced by a **positive charge** placed at that point in the field.

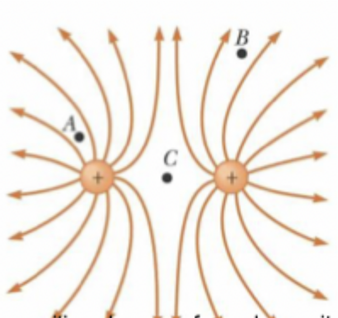


There are some rules for drawing electric field lines:

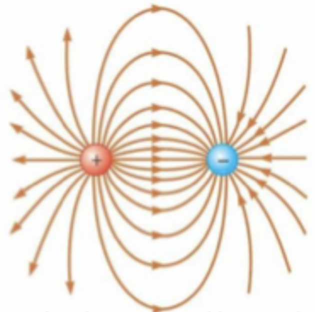
1. Field lines are directed **from positive charges to negative charges.**
2. The **number of field lines** drawn leaving a positive charge or approaching a negative charge should be **proportional to the magnitude of the charge.**

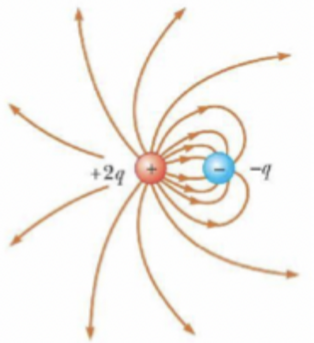
3. Field lines **cannot cross,** because the direction of $\boldsymbol{E}$ at any point must be unique.

Some examples of electric field lines drawn for some set-ups are as shown:

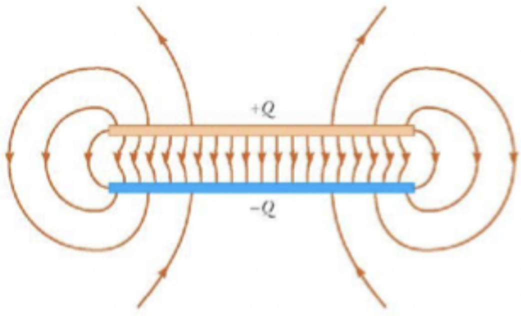


(c) Two positive charges of equal magnitude


(d) Two opposite charges equal in magnitude



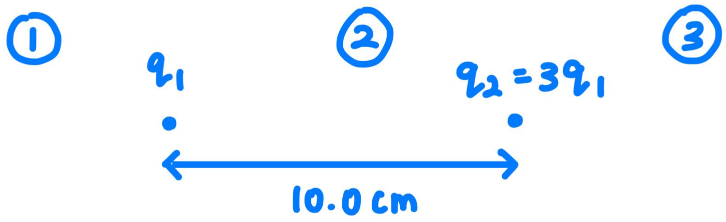
(e) Two opposite charges, the magnitude of one charge being double the other.



(f) A pair of oppositely charged parallel plates

Example 1.2. Two positively charged particles q_1 and $q_2 = 3q_1$ are 10.0 cm apart. Where (other than at infinity) could a third charge q_3 be placed so as to experience no net force?

Solution. The best way to approach these kinds of questions is to consider all the possible regions which the third charge could possibly lie in. There are three such regions, labelled 1, 2 and 3 in the diagram below:



Clearly, if we place any charge in either region 1 or 3, the electric force due to both q_1 and q_2 will be pointing in the same direction. There is hence no way for the electric forces to cancel out to produce no net force. As such, the only possibility is for q_3 to be placed in region 2.

Suppose we place q_3 at a distance of x to the right of q_1, where $x < 10.0\,\text{cm}$. Then, by equating the two electric forces, we have:

$$\frac{kq_1q_3}{x^2} = \frac{kq_2q_3}{(10.0-x)^2} \quad\Longrightarrow\quad \frac{1}{x^2} = \frac{3}{(10.0-x)^2} \quad\Longrightarrow\quad x = 3.66\,\text{cm}$$

where we rejected the negative root because $x > 0$. Hence, q_3 should be placed 3.66 cm from q_1, in between the two charges.

Example 1.3 (SJPO 2008)**.** Two point charges, Q_1 and Q_2, are placed in a vacuum at a distance of 0.200 m from each other. They attract each other with a force of 1.20 N. Now, the objects are brought into contact, and the net charge is shared equally, before they are returned to their initial positions. Now, the objects repel each other with a force whose magnitude is equal to that of the initial attractive force. What is the magnitude of the initial charge on each object?

Solution. Since the initial force is attractive, $Q_1 Q_2 < 0$ (i.e. they have different signs). Using Equation (1), initially, we have:

$$F = \frac{k\,|Q_1 Q_2|}{r^2} \quad\implies\quad |Q_1 Q_2| = \frac{Fr^2}{k} = \frac{(1.20)\,(0.200)^2}{8.99 \times 10^9} = 5.34 \times 10^{-12}\,\mathrm{C}$$

After contact, the objects share the net charge equally, hence they both now have a charge of $\frac{Q_1+Q_2}{2}$. Hence, using Equation (1), the repulsive force is now:

$$F = \frac{k}{r^2}\left(\frac{Q_1 + Q_2}{2}\right)^2$$

Since the magnitude of the attractive and repulsive force is the same, equating them, we get:

$$\left(\frac{Q_1 + Q_2}{2}\right)^2 = |Q_1 Q_2| \quad\implies\quad (Q_1 + Q_2)^2 = 4\,|Q_1 Q_2|$$

Because the initial charges were opposite, let's write their magnitudes as $a = |Q_1|$ and $b = |Q_2|$. Let's suppose $Q_1 = +a$ and $Q_2 = -b$. Then, we have the following simultaneous equations:

$$\begin{cases} (a-b)^2 = 4ab \\ ab = 5.34 \times 10^{-12} \end{cases}$$

You may solve this yourself as an exercise. You should obtain $a = 5.58 \times 10^{-6}\,\mathrm{C}$ and $b = 9.57 \times 10^{-7}\,\mathrm{C}$. As such, the initial charges are $Q_1 = +5.58\,\mu\mathrm{C}$ and $Q_2 = -0.957\,\mu\mathrm{C}$, or vice-versa for the signs.

1.1.3 Electric Potential & Potential Energy

Consider two point charges q_1 and q_2, located at a distance of r apart. The **electric potential energy (EPE)** U_E between the two charges is given by:

$$U_E = \frac{kq_1q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} \tag{4}$$

More formally, the electric potential energy is defined as the **work done by an external agent** to bring the charges from infinity to their respective positions in the system, without producing any acceleration on them (which would otherwise be "wasted" into kinetic energy). Take note that by convention, we take **infinity** to be the reference point, where there is 0 EPE.

Remark. Sometimes, the units for electric potential energy are written as eV (electron-volts), where $1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$.

The **electric potential** V_E can be thought of as the electric potential energy per unit charge. Hence, for a point charge Q, the electric potential V_E at a distance r away is given by:

$$V_E = \frac{kQ}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \tag{5}$$

As such, if a charge q is present in an electric field with a potential V_E, the electric potential energy U_E is given by:

$$U_E = qV_E \tag{6}$$

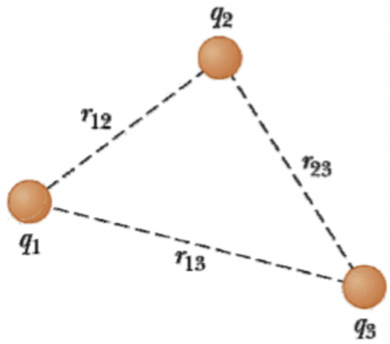
And, the work done by the external agent in moving a charge q from a potential V_A to potential V_B is given by:

$$U_{A\to B} = U_B - U_A = q\,(V_B - V_A) \tag{7}$$

Both the electric potential energy and electric potential are scalar quantities.

Remark. The units for electric potential are J/C, but it is more common to use V (volts).

Example 1.4. Find the electric potential energy of the system of three charges shown below.



Solution. A naive approach would be to consider the EPE associated with each charge, and them sum it all up. By doing so, you will get six different EPE terms (since each of the three charges interacts with two other charges).

However, this is **wrong!** The EPE between two charges is associated with the **interaction** between the two charges. It doesn't "belong" to any one of them. Hence, we need to be careful **not to double-count** the EPE terms!

Between three charges, there are three interactions. Hence, the total EPE is:

$$U_E = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$

Alternatively, you can also double-count first, but remember to include a factor of $\frac{1}{2}$ in the end.

Example 1.5. A proton is fired from far away at a 1.0 mm diameter glass sphere that has been charged to +100 nC and is fixed in position. What initial speed must the proton have to just reach the surface of the sphere? (The mass of the proton is $m_p = 1.67 \times 10^{-27}$ kg.)

Solution. Essentially, the KE of the proton is converted into the EPE of the interaction between the proton and the sphere. Hence, by COE, we have:

$$K_i = U_f \quad\Longrightarrow\quad \frac{1}{2}m_p v^2 = \frac{kq_sq_p}{r}$$

$$\Longrightarrow \quad v = \sqrt{\frac{2kq_pq_s}{m_pr}} = \sqrt{\frac{2\,(8.99\times10^9)\,(1.60\times10^{19})\,(100\times10^{-9})}{(1.67\times10^{-27})\left(\frac{1.0\times10^{-3}}{2}\right)}} = 1.86\times10^7 \text{ m/s}$$

1.1.4 Relationships Between Force, Field, Potential & Potential Energy

At this point, you should already be feeling some sense of *deja-vu*. Doesn't **electrostatics** feel so much like **gravitation?** All the formulas seem to be of a very similar form, somehow.

It does turn out that there is an analogy between electrostatics and gravitation! Essentially, charge is the analogue of mass. As such, when it comes to relating the basic four quantities, we should be expecting very similar things to gravitation.

Indeed, we have the following:

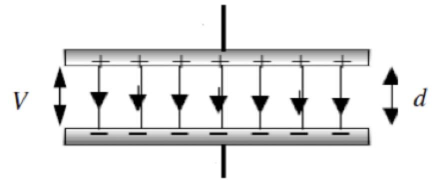
1. The negative of the gradient of the graph of U_E against r is F_E.
2. The negative of the gradient of the graph of V_E against r is E.
3. The negative of the area under the graph of F_E against r is U_E.
4. The negative of the area under the graph of E against r is V_E.

Do not forget the **negative signs!**

1.1.5 Parallel Plates

Consider two large metal plates that carry equal but opposite charges $+Q$ and $-Q$. This can be done by connecting them to opposite terminals of a constant voltage supply. We assume that within each plate, the charges are uniformly distributed.

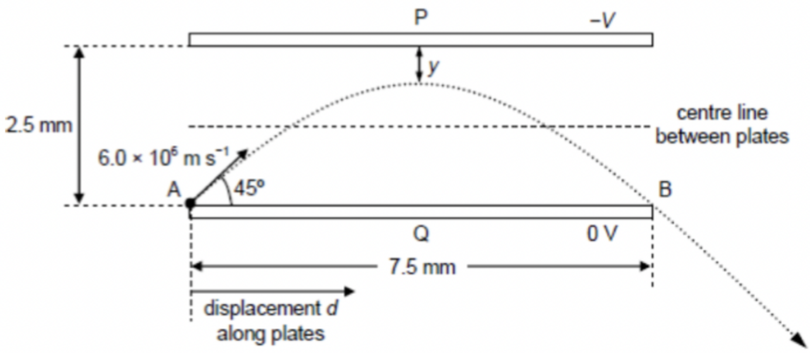
This set-up is able to create a **uniform electric field** as shown:



If the potential difference between the plates is V and they are separated by some **small** distance d, then the magnitude of the electric field E between them is given by:

$$E = \frac{V}{d} \tag{8}$$

Example 1.6. Two identical parallel conducting square plates P and Q have a length of 7.5 mm with a separation of 2.5 mm (assume this is small enough for the electric field to be uniform). The electric potentials of plates P and Q are $-V$ and 0 respectively. An electron enters the region between the plates at an angle of 45° with a speed of 6.0×10^6 m/s at point A and exits at point B as shown. (a) Calculate the time for which the electron is between the plates. (b) Find the potential difference across the plates. (c) Determine the electron's distance of closest approach, y, to plate P. (The mass of the electron is $m_\mathrm{e} = 9.11 \times 10^{-31}$ kg.)



Solution. (a) Notice that the electric field is uniform between the plates and only exists in the y-direction. As such, the electron experiences a uniform downward acceleration in the y-direction. This is akin to standard projectile motion, except we replace g with the acceleration associated with the electric force.

As such, the x-component of the electron's velocity is unchanged through the motion. Hence, the time taken is

$$t = \frac{x}{v_x} = \frac{x}{v\cos\theta} = \frac{7.5\times10^{-3}}{(6.0\times10^6)\cos45^\circ} = 1.77\times10^{-9}\,\text{s}$$

(b) The magnitude of the acceleration of the electron is given by:

$$a_y = \frac{F_E}{m_e} = \frac{eE}{m_e} = \frac{eV}{m_e d}$$

where $E = \frac{V}{d}$ is the electric field between the parallel plates. Note that even though there is a downward gravitational force, we ignore it because electric forces are a lot more significant. (You can do a quick calculation to verify this yourself.)

Considering the kinematics in the y-direction, from A to B, we have:

$$y = v_y t + \frac{1}{2}a_y t^2 \quad\Longrightarrow\quad 0 = (v\sin\theta)\,t - \frac{1}{2}\left(\frac{eV}{m_e d}\right)t^2$$

As such, the potential difference across the plates is

$$V = \frac{2vm_e d\sin\theta}{et} = \frac{2\,(6.0\times10^6)\,(9.11\times10^{-31})\,(2.5\times10^{-3})\sin45^\circ}{(1.60\times10^{-19})\,(1.77\times10^{-9})} = 68.3\,\text{V}$$

(c) The electron reaches its maximum height at a time $\frac{t}{2}$, where t is the time taken to get from A to B. Hence, the maximum height h is given by:

$$h = v_y\left(\frac{t}{2}\right) + \frac{1}{2}a_y\left(\frac{t}{2}\right)^2 = \frac{vt\sin\theta}{2} - \frac{eVt^2}{8m_e d}$$

$$= \frac{(6.0\times10^6)\,(1.77\times10^{-9})\sin45^\circ}{2} - \frac{(1.60\times10^{-19})\,(68.3)\,(1.77\times10^{-9})^2}{8\,(9.11\times10^{-31})\,(2.5\times10^{-3})} = 1.875\,\text{mm}$$

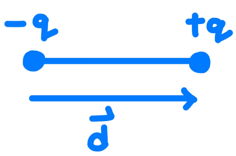
Hence, the distance of closest approach to plate P is $2.5 - 1.875 = 0.625\,\text{mm}$.

1.1.6 Electric Dipoles

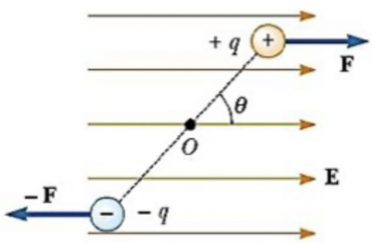
An electric dipole is a separation of positive and negative charges. Consider two charges $+q$ and $-q$ separated by a distance d. The **electric dipole moment $\boldsymbol{p}$** is given by:

$$\boldsymbol{p} = q\boldsymbol{d} \tag{9}$$

where $\boldsymbol{p}$ and $\boldsymbol{d}$ are defined to be pointing from the **negative charge to the positive charge.**



Now, let's consider an electric dipole in an **external, uniform electric field $\boldsymbol{E}$**.



The magnitude of the force F acting on each charge is $F = qE$, and they are opposite in direction. As such, the dipole experiences **no net force.** However, if $\theta \neq 0$ (as shown in the diagram above), then the dipole experiences **non-zero net torque!**

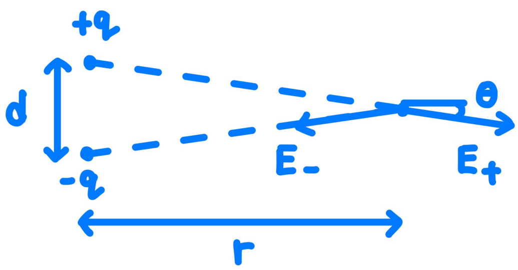
The magnitude of the torque $\boldsymbol{\tau}$ experienced by the dipole about its centre is given by:

$$\tau = 2F\left(\frac{d}{2}\sin\theta\right) = Fd\sin\theta = qdE\sin\theta = pE\sin\theta \tag{10}$$

Note that the torque always **decreases the angle** to bring it back to 0.

Example 1.7. Consider an electric dipole with a positive charge $+q$ located at $\left(0, \frac{d}{2}\right)$ and a negative charge $-q$ located at $\left(0, -\frac{d}{2}\right)$. The magnitude of the electric dipole moment is given by $p = qd$. (a) Find the electric field at $(r, 0)$. (b) Find the electric field at $(0, r)$. *Take $r \gg d$ for both cases, and employ the approximation $(1+x)^n \approx 1 + nx$ whenever possible for small x.*

Solution. (a) Let's first draw a diagram:



The magnitudes of the electric fields by each charge are:

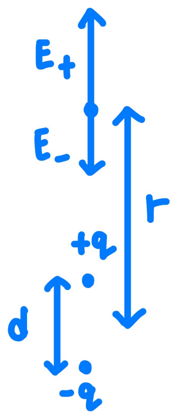
$$E = E_+ = E_- = \frac{kq}{r^2 + \left(\frac{d}{2}\right)^2}$$

The x-component of the electric fields cancel out. Hence, the net electric field points downwards along the y-axis, and is given by:

$$E_{\text{net}} = 2E\sin\theta = \left(\frac{2kq}{r^2 + \left(\frac{d}{2}\right)^2}\right)\left(\frac{\frac{d}{2}}{\sqrt{r^2 + \left(\frac{d}{2}\right)^2}}\right) = \frac{kqd}{\left(r^2 + \left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}} \approx \frac{kp}{r^3}$$

where we neglected the d^2 term because d is small.

(b) Let's draw another diagram:



The magnitudes of the electric fields by each charge are:

$$E_+ = \frac{kq}{\left(r - \frac{d}{2}\right)^2}, \quad E_- = \frac{kq}{\left(r + \frac{d}{2}\right)^2}$$

The former acts upwards while the latter acts downwards, both along the y-axis. Hence, the net electric field points upwards along the y-axis, and is given by:

$$E_{\text{net}} = E_+ - E_- = \frac{kq}{\left(r - \frac{d}{2}\right)^2} - \frac{kq}{\left(r + \frac{d}{2}\right)^2} = \frac{kq}{r^2}\left(\left(1 - \frac{d}{2r}\right)^{-2} - \left(1 + \frac{d}{2r}\right)^{-2}\right)$$

$$\approx \frac{kq}{r^2}\left(\left(1 + \frac{d}{r}\right) - \left(1 - \frac{d}{r}\right)\right) = \frac{2kqd}{r^3} = \frac{2kp}{r^3}$$

where we made use of the provided approximation since $\frac{d}{r} \ll 1$.

Interestingly, the electric field along the y-axis is twice of the electric field along the x-axis for the same distance from the origin.

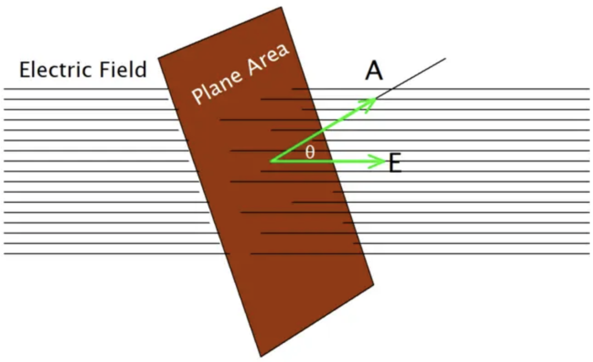
1.1.7 Electric Flux & Gauss' Law

Remark. While Gauss' Law is not in the SJPO syllabus, do note that recent SJPOs (2024 and 2025) have included problems on Gauss' Law (included in the problems behind as well).

Consider an electric field of magnitude E passing through a flat surface of area A. Let the (acute) angle between the electric field and the **normal to the surface** be θ.

The **electric flux** Φ_E through the surface is given by:

$$\Phi_E = EA\cos\theta \tag{11}$$



You can think of electric flux to be related to the number of field lines passing through the area.

Gauss' Law gives the relationship between the electric flux through a **closed surface** (a surface that encloses a finite volume) to the charge q enclosed by the surface:

$$\Phi_E = \frac{q}{\varepsilon_0} \tag{12}$$

Remark. Conventionally, for a closed surface, we assign **outward flux as positive** and **inward flux as negative.**

Gauss' Law is most commonly applied when there are **symmetries** in the set-up that we can exploit. Most commonly, these are:

1. Spherical symmetry
2. Cylindrical symmetry
3. Infinite plane symmetry

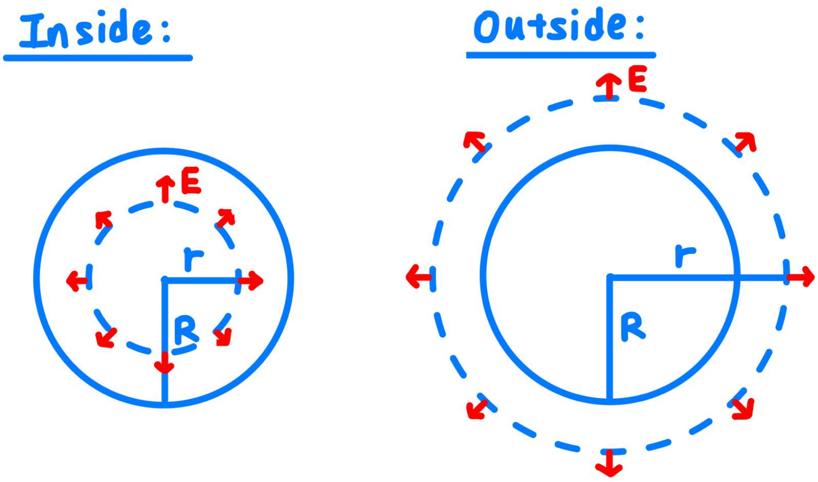
The idea here is to construct an appropriate closed surface to evaluate the electric flux over, as per Equation (11), and then link it to the total charge enclosed by the surface as per Equation (12). This is what we call a **Gaussian surface.**

A good choice of Gaussian surface respects the symmetry of the set-up. It is usually one whereby the electric field is **uniform over the whole surface** and is **perpendicular to the whole surface,** which allows us to evaluate the electric flux very easily.

The **three** most common Gaussian surfaces are the **sphere, cylinder and pillbox.** The following three examples will show how to use them.

Example 1.8. Consider a sphere of radius R and total positive charge Q distributed uniformly across its volume. Find the electric field (magnitude and direction) everywhere.

Solution. In this case, a good choice of Gaussian surface would be a **sphere,** because it respects the spherical symmetry of the set-up.



For the case of inside the sphere ($r < R$), the enclosed charge is given by $q = \frac{r^3}{R^3}Q$, by using the ratio of volumes. Hence, by Gauss' Law, we have:

$$\Phi_E = \frac{q}{\varepsilon_0} \implies E\left(4\pi r^2\right) = \frac{Qr^3}{\varepsilon_0 R^3} \implies E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

For the case of outside the sphere ($r \geq R$), the enclosed charge is just $q = Q$. Hence, by Gauss' Law, we have:

$$\Phi_E = \frac{q}{\varepsilon_0} \implies E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \implies E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

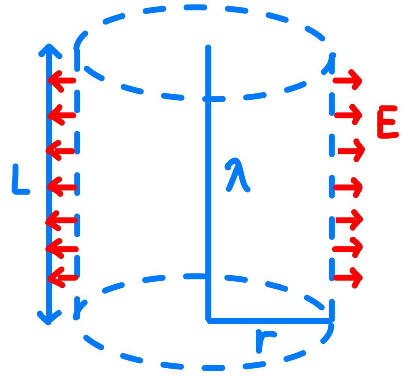
Hence, as a piecewise function, we can write:

$$E = \begin{cases} \frac{Qr}{4\pi\varepsilon_0 R^3}, & r < R \\ \frac{Q}{4\pi\varepsilon_0 r^2}, & r \geq R \end{cases}$$

In any case, the direction of the electric field is radially outwards from the centre of the sphere.

Example 1.9. Consider a long, straight wire with uniform positive linear charge density (charge per unit length) λ. Find the electric field (magnitude and direction) at a distance r away.

Solution. In this case, a good choice of Gaussian surface would be a **cylinder,** because it respects the cylindrical symmetry of the set-up.



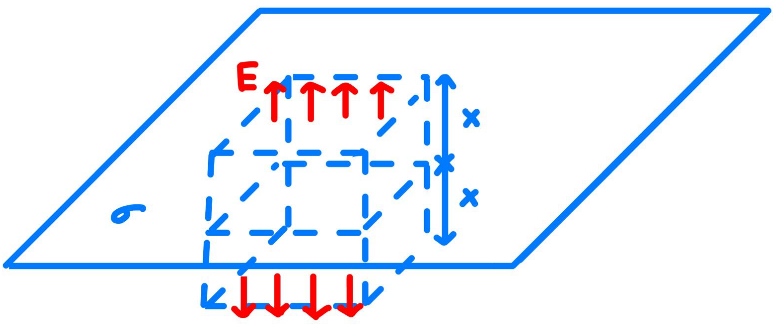
Considering a length L of the wire and cylinder, the enclosed charge is $q = \lambda L$. Hence, by Gauss' Law, we have:

$$\Phi_E = \frac{q}{\varepsilon_0} \quad \Longrightarrow \quad E\left(2\pi r L\right) = \frac{\lambda L}{\varepsilon_0} \quad \Longrightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The direction of the electric field is radially outwards from the wire.

Example 1.10. Consider a large, thin conducting plate with uniform positive area charge density (charge per unit area) σ. Find the electric field (magnitude and direction) at a distance x away.

Solution. In this case, a good choice of Gaussian cylinder would be a **pillbox,** because it respects the infinite planar symmetry of the set-up.



Considering an area A of the plane and pillbox, the enclosed charge is $q = \sigma A$. Hence, by Gauss' Law, we have:

$$\Phi_E = \frac{q}{\varepsilon_0} \quad \Longrightarrow \quad 2EA = \frac{\sigma A}{\varepsilon_0} \quad \Longrightarrow \quad E = \frac{\sigma}{2\varepsilon_0}$$

The direction of the electric field is directed away from the plane. Surprisingly, the electric field is uniform and independent of the distance x from the plane!

1.2 Conductors

A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium. As we shall see, a conductor in **electrostatic equilibrium** (no further movement of charges) has the following key properties:

1. **The electric field is zero everywhere inside the conductor.** If the field is not zero, free charges in the conductor would accelerate under the action of the field. This motion of electrons, however, would mean that the conductor is not in electrostatic equilibrium.

2. **If an isolated conductor carries a charge, it resides on its surface.** Electric field must be zero at every point inside a conductor, hence, any net charge on the conductor must reside on its surface. For a more in-depth discussion with Gauss's Law, see the appendix.

3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor. This can be explained by the fact that if there were any component of the electric field tangential to the surface, it would cause further redistribution of charge.

4. **A conductor must be equipotential (same potential everywhere).** Consider two points A and B on the surface of a charged conductor. Along a surface path connecting these points, $\mathbf{E}$ is always perpendicular to the displacement $\mathrm{d}\mathbf{s}$; therefore $\mathbf{E} \cdot \mathrm{d}\mathbf{s} = 0$. It means that the potential difference between A and B is necessarily zero:

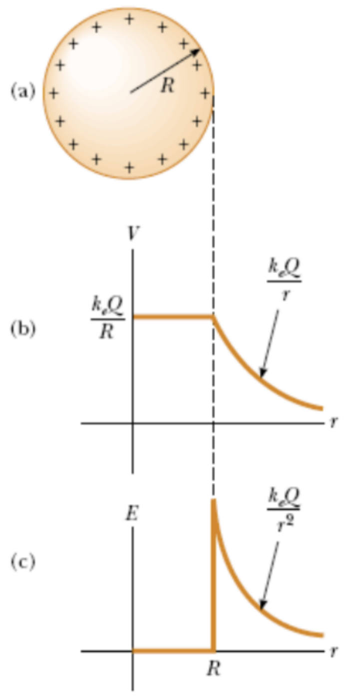
$$V_B - V_A = -\int_A^B \mathbf{E} \cdot \mathrm{d}\mathbf{s} = 0$$

   This result applies to any two points on the surface. Therefore, V is constant everywhere on the surface of a charged conductor in equilibrium.

   Without calculus, to think about it, the electric field is always perpendicular to the surface of conductor, so it means that there is no electric field components parallel to the surface, as such there should not be any potential difference between two points on the surface of conductor.

Example 1.11. Draw the graphs of electric field and electric potential against distance from the centre of a positively charged, isolated conductive sphere.

Solution. The answer is



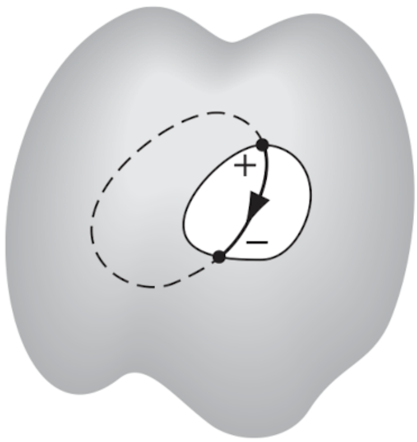
We know that $E = 0$ within the sphere from property (1). Outside the sphere, because its isolated, it behaves just like a point charge, explaining graph (c). Recall that because electric field is the negative of the gradient of electric potential $E = -\frac{dV}{dr}$, the change in potential must be zero within the sphere, thus it is a constant. Outside of the sphere, it once again behaves just like a point charge, and asymptotes to zero as you approach infinity.

Example 1.12. (Electrostatic Shielding) You might have heard of a common saying to hide in a car if you are unable to get under shelter when there is lightning outside. Why is that safe?

Solution. Suppose we have a conductor of arbitrary shape with a cavity (a region of empty space) in it. This resembles a car which we assume to be made of conductive material and has a empty space for you to sit inside. If there is no charge in the cavity, but there could be charges elsewhere outside of the conductor (causing the lightning), it turns out that the electric field in the cavity is actually zero everywhere inside the cavity.

To show this, we start by reminding ourselves of property 1. Now suppose, for contradiction, that the electric field inside the empty cavity were *not* zero.

Then there would have to be electric field lines inside the cavity. But electric field lines begin on positive charge and end on negative charge, so those field lines would have to start and end on charges located on the cavity wall.



That would mean there is a + region and a − region on the inner surface of the cavity.

Now choose one such field line inside the cavity, going from the positive patch to the negative patch. Then complete it into a closed loop by returning through the conducting material.

- Along the part of the loop inside the cavity, $\mathbf{E} \cdot d\mathbf{l} > 0$ if we move along the field line.
- Along the part of the loop inside the conductor, $\mathbf{E} = 0$, so that part contributes nothing.

So for the whole closed loop we would get

$$\oint \mathbf{E} \cdot d\mathbf{l} > 0.$$

But in electrostatics,

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

for every closed loop[1], so our assumption was wrong and the electric field

$$\mathbf{E} = 0 \text{ in an empty cavity}$$

If the electric field in the cavity is zero, then **there cannot be any charge on the inner wall of the cavity either**.

So in conclusion, the conductor *shields* the cavity from external electric fields. Charges in the conductor rearrange themselves so that the electric field inside the metal is zero, and as a result an empty enclosed cavity also ends up with zero field. This is the basic idea behind a **Faraday cage**: a conducting enclosure protects its interior from outside electric fields.

[1]Electric field is a vector, just like displacement: imagine starting at point and walking a giant loop back to the same point – your net displacement is zero!

2 Problems

Problem 2.1 (SJPO 2014)**.** A system of 1614 particles, each of which is either an electron or a proton, has a net charge of -4.544×10^{-17} C. What is the total mass of this system?

(A) 1.11×10^{-24} kg

(B) 1.35×10^{-24} kg

(C) 1.47×10^{-27} kg

(D) 1.59×10^{-27} kg

(E) 2.70×10^{-27} kg

Solution. (A)

Problem 2.2 (SJPO 2019)**.** A metal sphere has a diameter of 60.0 cm. It is charged until its surface charge density is $12.00\,\mathrm{nC/m^2}$. What is the electric potential at the surface of the sphere?

(A) 122.0 V

(B) 339.0 V

(C) 406.8 V

(D) 488.1 V

(E) 813.5 V

Solution. (C)

Problem 2.3 (SJPO 2014)**.** In a Millikan oil drop experiment, an oil drop of mass 1.2×10^{-5} kg is held stationary between a pair of parallel plates held at 2.0 cm apart. The potential difference across the plates is 120 V. Assume the distance between the plates is small enough so that the electric field in between is uniform. What is the charge on the oil droplet?

(A) 1.6×10^{-19} C

(B) 6.4×10^{-19} C

(C) 2.0×10^{-18} C

(D) 4.9×10^{-15} C

(E) 9.8×10^{-17} C

Solution. (C)

Problem 2.4 (SJPO 2017)**.** Simplifying Thomson's plum pudding model, consider a gold nucleus as a sphere with radius 1.44×10^{-10} m and charge $+79e$ uniformly distributed within the volume of the sphere. The gold nucleus is fixed in position. There is also a helium nucleus of charge $+2e$, which can be considered as a point charge, which approaches the gold nucleus from very far away. Considering only electrostatics, how much kinetic energy does the helium nucleus need to have initially, when it was far away, in order to just pass through the gold nucleus in a head-on collision?

(A) 1.0 keV

(B) 1.2 keV

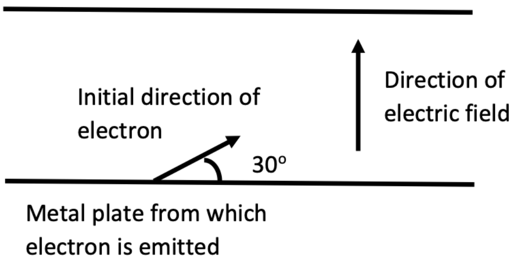
(C) 2.4 keV

(D) 3.6 keV

(E) 4.7 keV

Solution. (C)

Problem 2.5 (SJPO 2016)**.** An electron is emitted from the surface of a metal plate at an angle of 30° from the surface. The electron's initial kinetic energy is 3.2×10^{-19} J. A uniform electric field of 1000 N/C is applied, direction as shown in the diagram below. Considering only electrical effects, what is the kinetic energy of the electron when it is furthest from the plate from which it was emitted?



(A) 0 J

(B) 0.8×10^{-19} J

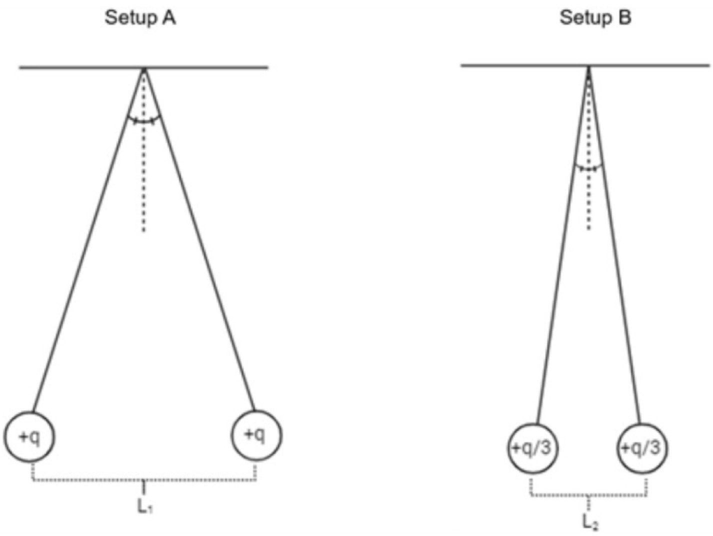
(C) 1.6×10^{-19} J

(D) 2.4×10^{-19} J

(E) 3.2×10^{-19} J

Solution. (D)

Problem 2.6 (SJPO 2024, unused)**.** In setup A, two identical small spheres of the same mass and charge are in equilibrium at a distance of L_1 from each other when suspended by very long, light and insulating strings. In setup B, the same spheres are suspended from the same strings but now each only have one-third the charge, and settle into equilibrium at a distance L_2 from each other. What is the ratio $\frac{L_1}{L_2}$?



(A) $3\sqrt{2}$

(B) 4

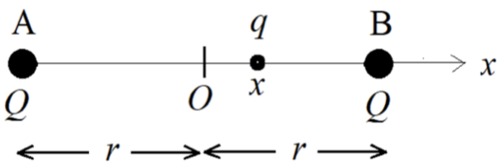
(C) $\sqrt[3]{9}$

(D) 3

(E) $\sqrt[3]{4}$

Solution. (C)

Problem 2.7 (SJPO 2022)**.** Two fixed positive point charges Q are at a distance of $2r$ apart, one at A and one at B, as per the diagram below. Let the mid-point between them be the origin of the x-axis. A third positive charge q of mass m is placed at a point a distance x away from the origin. You may first find it helpful to determine the net force on q when it is a distance x away from the origin. What is the time taken for the charge q to first reach the origin? *Hint: You might find the identity $a^2 - b^2 = (a+b)(a-b)$ useful. The term $\frac{x^2}{r^2}$ is very small compared to 1, and so may be ignored, i.e. $1 \pm \frac{x^2}{r^2} \approx 1$.*



(A) $\frac{\pi r}{4}\sqrt{\frac{mr}{kQq}}$

(B) $\frac{\pi r}{2}\sqrt{\frac{mr}{kQq}}$

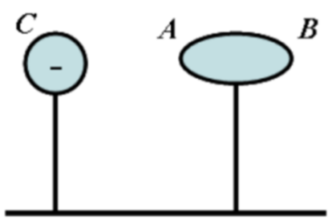
(C) $\frac{\pi}{4r}\sqrt{\frac{kQq}{mr}}$

(D) $\frac{\pi}{2r}\sqrt{\frac{kQq}{mr}}$

(E) $\frac{\pi x}{2}\sqrt{\frac{mr}{kQq}}$

Solution. (A)

Problem 2.8. As shown in the figure, a neutral conductor is mounted on an insulator and brought close to a negatively charged conductor C. When electrostatic equilibrium has been achieved, which of the following mathematical statements regarding the potential of the A and B ends of the conductor is true? (Assume ground to be of zero potential.)



(A) $V_A > V_B$

(B) $V_A = V_B = 0$

(C) $V_A = V_B > 0$

(D) $V_B > V_A$

(E) $V_A = V_B < 0$

Solution. (E)

Problem 2.9. (SJPO 2011) Two identical small conducting spheres are separated by 0.60 m. The spheres carry different amounts of charge and each sphere experiences an attractive electric force of 10.8 N. The total charge on the two spheres is $-24\ \mu$C. The two spheres are connected by a thin conducting wire, which is then removed. The electric force on each sphere is closest to:

(A) Zero

(B) 3.6 N, repulsive

(C) 3.6 N, attractive

(D) 5.4 N, repulsive

(E) 5.4 N, attractive

Solution. (B)

Problem 2.10. A neutral conducting block has four empty spaces, J, K, L, M within it. An external charge of $+3q$ is now placed in space J. Three other charges are placed in spaces K, L, M. A charge of $+6q$ has now collected on the outer surface of the block. Which of the following combination of charges for spaces K, L, M is possible?

(a) $-q,\ +3q,\ +2q$

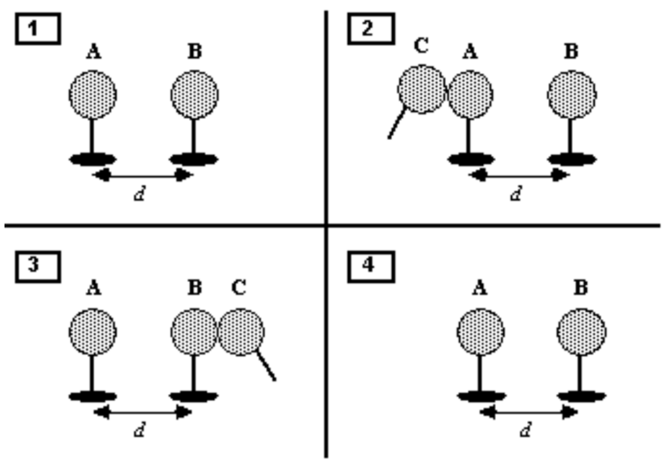
(b) $+q,\ -2q,\ +4q$

(c) $-3q,\ +2q,\ +3q$

(d) $-3q,\ +2q,\ +q$

Solution. (B)

Problem 2.11. Two identical conducting spheres A and B carry equal amounts of excess charge that have the same sign (Frame 1). They are separated by a distance d (which may be assumed to be large compared to the dimension of the spheres). Sphere A exerts an electrostatic force on sphere B that has a magnitude F. Another identical sphere C, on an insulating rod and is uncharged, is touched first to sphere A (Frame 2) and then sphere B (Frame 3) and is finally removed (Frame 4). The magnitude of the electrostatic force that sphere A exerts on sphere B in Frame 4 is approximately



(A) zero.

(B) $3F/8$.

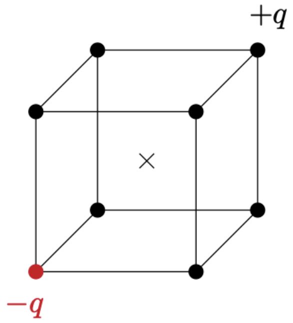
(C) $F/3$.

(D) $F/2$.

(E) $3F/4$.

Solution. (B)

Problem 2.12 (SJPO 2024)**.** Consider a cube of side length a. 7 identical charges $+q$ and one additional charge $-q$ are placed on vertices of the cube, as shown below. Find the electric field at the centre of the cube.



(A) 0

(B) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}$

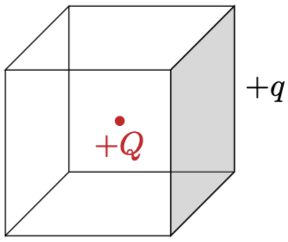
(C) $\frac{1}{4\pi\varepsilon_0}\frac{4q}{a^2}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{4q}{3a^2}$

(E) $\frac{1}{4\pi\varepsilon_0}\frac{8q}{3a^2}$

Solution. (E)

Problem 2.13 (SJPO 2024)**.** Consider a cube of side length a. A charge $+q$ is spread uniformly across one of the insulating faces of the cube. A charge $+Q$ is placed at the centre of the cube. Find the electric force exerted on the charge $+Q$.



(A) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{6a^2}$

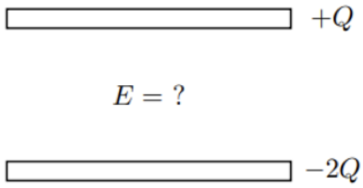
(C) $\frac{1}{4\pi\varepsilon_0}\frac{2Qq}{3a^2}$

(D) $\frac{1}{\varepsilon_0}\frac{Qq}{a^2}$

(E) $\frac{1}{\varepsilon_0}\frac{Qq}{6a^2}$

Solution. (B)

Problem 2.14 (SJPO 2025)**.** Consider two metal plates that are oppositely charged and parallel with equal areas A. One plate contains charge $+Q$ while the other plate contains charge $-2Q$. Assuming that the separation between the plates is small, what is the electric field between the two plates?



(A) $\frac{1}{2}\frac{Q}{\varepsilon_0 A}$

(B) $\frac{Q}{\varepsilon_0 A}$

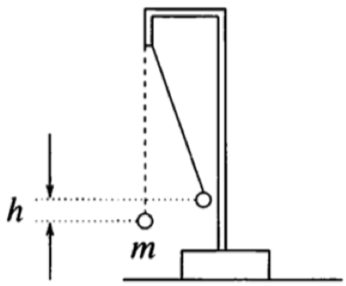
(C) $\frac{3}{2}\frac{Q}{\varepsilon_0 A}$

(D) $2\frac{Q}{\varepsilon_0 A}$

(E) $3\frac{Q}{\varepsilon_0 A}$

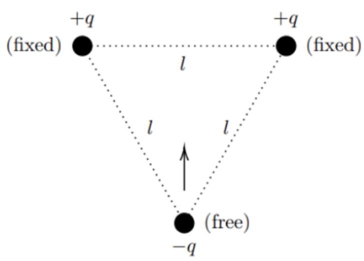
Solution. (C)

Problem 2.15 (200 Puzzling Physics Problems)**.** A small positively charged ball of mass m is suspended by an insulating thread of negligible mass. Another positively charged ball is moved very slowly from a large distance until it is in the original position of the first ball. As a result, the first ball rises by height h. How much work has been done? Leave your answer only in terms of m, h and any constants.



Solution. GPE+EPE=3mgh

Problem 2.16 (SJPO 2025)**.** Three charges of mass m are initially arranged to form an equilateral triangle of side l, with two fixed $+q$ charges and one free $-q$ charge. The $-q$ charge is now released from rest, and it begins to undergo oscillatory motion. What is its maximum speed?



(A) $\frac{q}{\sqrt{\pi\varepsilon_0 m l}}$

(B) $\frac{1}{2}\frac{q}{\sqrt{\pi\varepsilon_0 ml}}$

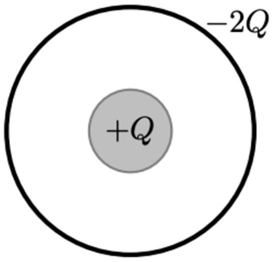
(C) $\frac{1}{\sqrt{2}}\frac{q}{\sqrt{\pi\varepsilon_0 ml}}$

(D) $\frac{\sqrt{3}}{2}\frac{q}{\sqrt{\pi\varepsilon_0 ml}}$

(E) $\sqrt{\frac{\sqrt{3}}{2}}\frac{q}{\sqrt{\pi\varepsilon_0 ml}}$

Solution. (A)

Problem 2.17. (SJPO 2024) A conducting sphere of charge $+Q$ and radius a is placed in the centre of a spherical conducting shell of charge $2Q$ and radius b. What is the minimum work required to bring a small charge $+q$ from the outer shell to the surface of the inner sphere?



(A) $\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

(B) $\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)$

(C) $\frac{Qq}{4\pi\varepsilon_0 a}$

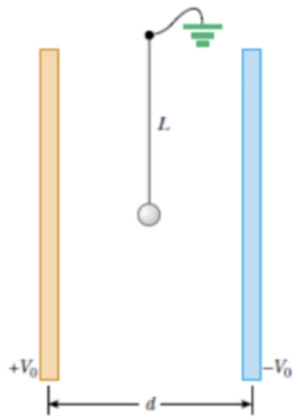
(D) $\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{2}{b}\right)$

(E) $\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{a}+\frac{2}{b}\right)$

Solution. (B)

Problem 2.18. As shown below, two large parallel vertical conducting plates separated by distance d are charged so that their potentials are $+V_0$ and $-V_0$. A small conducting ball of mass m and radius R (where $R \ll d$) is hung midway between the plates. The thread of length L supporting the ball is a conducting wire connected to ground, so the potential of the ball is fixed at $V = 0$. The ball hangs straight down in stable equilibrium when V_0 is sufficiently small. Show that the equilibrium of the ball is unstable if V_0 exceeds the critical value:

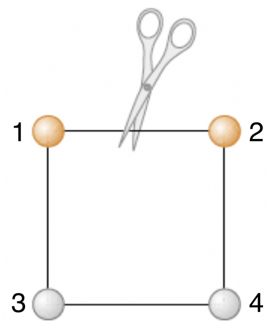
$$V_{\max} = \sqrt{\frac{d^2 mg}{16\pi\varepsilon_0 RL}}$$



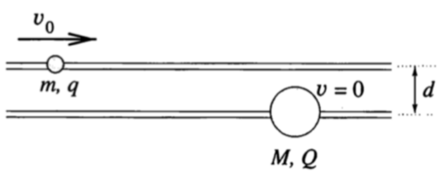
Hint: Stable EQ means that a slight displacement can be adjusted back by a restoring force

Problem 2.19 (200 Puzzling Physics Problems)**.** Two positrons (same mass as an electron but with charge $+e$) are at opposite corners of a square of side a. The other two corners of the square are occupied by protons. All particles have charge q, and the proton mass M is much larger than the positron mass m. Find the approximate speeds of the particles much later.

Problem 2.20. Four balls, each of mass m, are connected by four non-conducting strings to form a square with side length a, as shown below. The assembly is placed on a horizontal non-conducting frictionless surface. Balls 1 and 2 each have charge q, and balls 3 and 4 are uncharged. Find the maximum speed of balls 1 and 2 after the string connecting them is cut.



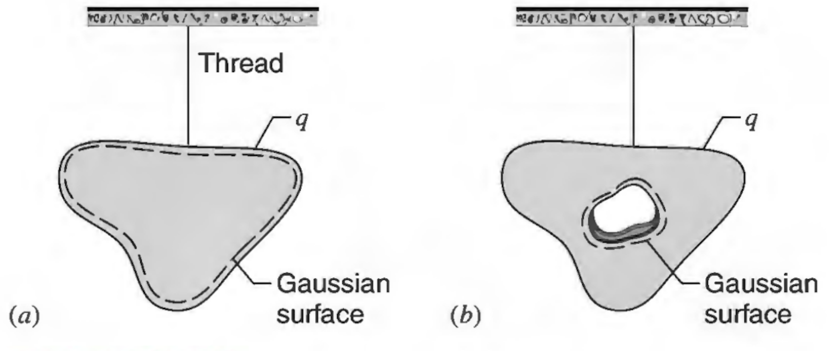
Problem 2.21 (200 Puzzling Physics Problems)**.** Two small beads slide without friction, one on each of two long horizontal parallel fixed rods a distance d apart. The masses of the beads are m and M and they carry charges q and Q respectively. Initially, the larger mass M is at rest and the other one is far away approaching it at a speed v_0. Find the minimum value of v_0 so that the smaller bead can ever get to the right of the larger bead.



3 Appendix

3.1 Understanding Conductors using Gauss's Law

We can more formally prove the 2nd property of conductors presented at the beginning of this set of notes by using Gauss's Law. Imagine drawing a Gaussian surface completely *inside* the conductor (case A in diagram below).



Since the electric field is zero everywhere on that surface,

$$\oint \mathbf{E} \cdot d\mathbf{A} = 0.$$

By Gauss' law, this means the net charge enclosed must be zero. So there cannot be any excess charge trapped inside the bulk of the conductor. Thus,

Any excess charge on an isolated conductor must be on its outer surface.

Note that the term **isolated conductor** is important because means a conductor that is not being influenced by external connections or devices. For example, a wire connected to a battery is *not* isolated, because charges are being driven through it and current flows.

So the points made in this section apply only to conductors in **electrostatic equilibrium**, not to conductors carrying current. Now suppose the conductor contains an empty cavity. A natural question is: *does charge appear on the inner wall of the cavity?*

If there is **no charge inside the cavity**, then the answer is no. Again, draw a Gaussian surface in the conducting material just around the cavity wall (case B in the diagram above now). Since the electric field inside the conductor is still zero, Gauss' law says the net enclosed charge must be zero. Since there is no charge inside the cavity, there can also be no charge on the inner surface. Thus,

If an isolated conductor has an empty cavity, there is no charge on the inner surface.

All the excess charge remains on the **outer surface**.

Now suppose a charge q' is placed inside the cavity. The electric field inside the *conducting material* must still remain zero. So the conductor responds by redistributing its charges. To make the net enclosed charge zero for a Gaussian surface drawn in the conductor around the cavity, the inner surface must acquire charge $-q'$. This is called an **induced charge**.

If the conductor originally had total charge q, then the outer surface must carry the rest:

$$q + q'.$$

This keeps the total charge of the conductor equal to its original value q. So in this case:

$$\boxed{\text{charge inside cavity } q' \Rightarrow \text{ inner surface gets } -q'}$$

and

$$\boxed{\text{outer surface gets } q + q'.}$$

The moral of the story is that the conductor itself does not magically “force” the field to be zero. What really happens is that the **free charges move until they reach exactly the arrangement needed to make the electric field zero inside the conducting material.**